Sum of Non-Concave Utilities Maximization for MIMO Interference Systems

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***Abstract*—To evaluate system capacity, past works on Multiple-Input-Multiple-Output (MIMO) systems with mutually interfering links have focused on maximizing the sum of mutual information as the objective criterion. Since the ultimate goal of a MIMO system is to support network applications used by consumers, we consider the non-concave sigmoid utility function, which is the recommended choice according to Shenker for modeling consumer satisfaction in applications with inelastic network traffic. We formulate the sum of utilities maximization as a global optimization problem with polynomial constraints and a rational objective function. Using a technique known as moment relaxation, we derive a sequence of Semidefinite Programming (SDP) problems whose optimal objective values converge to the global maximum sum of utilities. In our simulation examples, we employ our optimization model to determine the average global maxima sum of utilities by optimizing the covariance matrices of the transmitters. We then compare the results with those attainable by the alternative non-uniform optimal power control model that optimizes only the eigenvalues of the covariance matrices. By examining performance differences between the two models, we obtain insights about how interference and excessive data-rate requirements imposed by the application can impede link-consumers' ability to maximize their sum of utilities.**

***Index Terms*—MIMO, non-concave sigmoid utility, global optimization of rational function, moment relaxation, semidefinite programming.**

I. Introduction

EVER since the pioneering works of Foschini and Gans [1] and Telatar [2] that promise improved spectral efficiency using MIMO designs for isolated single-user wireless links, there has been extensive research on evaluating the throughput performance of systems consisting of multiple simultaneously transmitting MIMO links under the detrimental effects of co-channel interference. For example, interference from adjacent cells in MIMO cellular systems degrades the overall system capacity significantly [3], [4]. The same phenomenon arises in MIMO ad-hoc networks, where each TX (transmitter) and RX (receiver) pair suffers from interference imposed by other pairs operating in the same frequency band [5]. We henceforth refer to a system of mutually interfering MIMO links as a MIMO interference system.

An objective criterion for throughput performance is the system capacity, which involves maximizing the sum of mutual information of the network links. Computing capacity requires optimizing the input covariance matrices of the TXs. This is equivalent to choosing orthonormal eigenvectors spanning the transmission signal space and the corresponding non-negative eigenvalues for power allocation along each eigenvector axis. Since channel matrices form a key ingredient in the sum of mutual information formulation, the extent to which one can optimize the input covariance matrices to achieve higher capacity depends on the availability of Channel State Information (CSI) at the TXs[1] [6], [7]. In the absence of CSI, the TX has no knowledge of the specific realization of the channel matrices, and therefore is unable to fully optimize its input covariance matrix. That is, the absence of CSI renders the TX incapable of choosing any favourable eigen-subspace for interference avoidance [6], [7]. On the other hand, presence of CSI means the TX can capitalize on the known realization of the channel matrices to fully optimize its input covariance matrix, thereby utilizing its multiple antennas more effectively for interference avoidance to achieve higher capacity [6], [7].

There have been several works that compute the system capacity of a MIMO interference system under different assumptions regarding the availability of CSI at the TXs. When CSI is absent at the TXs, Telatar in Lemma 5 of [2] states that the channel matrix in the sum of mutual information formulation has the same distribution as the channel matrix being eigen-transformed by the unitary matrix derived from the eigen-decomposition of the covariance matrix input. Through this property, Blum in [8] proposes a power allocation model for computing the system capacity by optimizing only the eigenvalues.

Under the different assumption that CSI is present at the TXs, Liu *et al.* in [9] propose a sum of mutual information maximization formulation that assumes equal power allocation across all antennas in each TX node. However such an assumption may not hold in general.

In this paper, we focus on MIMO interference systems with feedback for flat fading channels similar to the one in [10] where CSI is available at the TXs and RXs. We also assume each RX performs single-user detection, meaning all unintended transmissions are treated as pure interference such that interference cancellation at the RX does not apply [10], [6]. Our focus on such systems is motivated by the fact that having CSI at the TXs enables the system to achieve higher sum of mutual information by optimizing the input covariance matrices of the TXs for each realization of the channel matrices.

Although maximizing the sum of mutual information is pertinent from the system performance point of view, achieving consumer satisfaction is also an important evaluation criterion based on the sum of utilities of consumers behind

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[1]The instantaneous realization of the channel matrices are known to the TXs.

the network links [11]. Depending on the choice of the utility function, an increase in the sum of mutual information may not yield a proportionate improvement in the sum of utilities. As a result, maximizing solely the sum of mutual information may overlook the important goal of improving the sum of utilities of consumers. Moreover, achieving good consumer satisfaction requires one to maximize the sum of utilities based on suitable utility functions that reflect reliably the experiences of consumers using the network application. Shenker [11] advocates that utilities for inelastic traffic are more accurately modeled as non-concave functions such as the sigmoid function. There are recent developments that employ the sigmoid utility function, such as power control and scheduling of single antenna systems [12], and utility maximization of MIMO broadcast channels [13].

In this paper, we offer the following contributions. We aim to compute the global maximum sum of utilities for a MIMO interference system supporting applications with inelastic network traffic. We first model a link consumer's utility as a sigmoid function [11] of the link's mutual information. We then give a problem formulation for computing the instantaneous global maximum sum of utilities given a specific realization of the stochastic channel matrices. In our model, we optimize the covariance matrices where CSI is assumed available at the TXs. Although our optimization model produces global maxima sum of utilities, we highlight that non-global optimal algorithms such as beamforming [6] are more likely to be used in real-world applications for practical reasons. In order to determine how close or how far the link-consumers operate relative to the global maxima sum of utilities under the negative effects of interference and excessive data-rate requirements, we compare our global maxima results with output coming from an alternative optimization model that does not fully optimize the covariance matrices. For comparison purpose, we refer to the optimal power control problem [9] but without the equal power allocation condition. To arrive at a non-uniform optimal power control model, we reuse our global optimization model but restricting the eigenvectors spanning the transmission signal space to unit orthonormal vectors. That is, the alternative model for comparison purpose is a restricted version of our model that optimizes only the eigenvalues. We then compute and compare the average global maxima sum of utilities achievable by our first model with the results attainable by the alternative model. In our simulation examples, we consider consumers with low and high data-rate needs who are using a network application facilitated by the MIMO interference system. By placing the link nodes within box areas of varying lengths, the links are subject to interference of different intensities.

In our optimization model, we express the sum of utilities maximization as a global optimization problem consisting of polynomial constraints and a rational objective function expressible as a ratio of two polynomials. To solve for the instantaneous global optimal objective value, we investigate the use of the *moment* approach [14], [15], [16] to reformulate the global optimization problem in the moment form. We then obtain moment relaxations expressible as linear SDP problems, solvable using SDP tools such as [17].

A. Related Work

In [9], Liu *et al.* compute the global maximum sum of mutual information for MIMO interference systems using Branch and Bound (BB) method with Reformulation Linearization Technique (RLT) [18] assuming CSI is available at the TXs. The BB-RLT approach solves linear RLT relaxations of the original maximization problem for performance bounds within an iterative BB framework. Due to our focus on optimizing the input covariance matrices in the transmission signal space under the assumption that CSI is available at the TXs, we represent the input covariance matrix of each TX as a Hermitian covariance matrix. Since the Hermitian inputs are positive semidefinite matrices [10], any future work undertaking RLT to compute the global maximum sum of mutual information will have to take into consideration the positive semidefinite constraints required for defining the input Hermitian covariance matrices of the TXs. Unfortunately, linear constraints on the RLT variables in the RLT relaxation alone are unable to describe accurately the positive semidefinite cones [19], [20].

Qian *et al.* in [21] use monotonic optimization [22] to achieve weighted throughput maximization for power control applications. In [12], Qian and Zhang extend their work to power control with scheduling to maximize system performance across time. However, monotonic optimization requires assumptions such as normality[2] of the feasible set, which is violated by positive semidefinite cones.

In [13], Brehmer and Utschick employ non-concave utilities to model applications with inelastic traffic in a MIMO broadcast channel (MIMO-BC) setup. The authors however assume normality of the feasible set that relies on the inherent convexity property of the capacity region of a MIMO-BC [23]. However, the same convexity property is not guaranteed to hold for a general MIMO interference system.

B. Contributions

The contributions of this paper consist of the following.

1) We formulate the global maximization of the sum of sigmoid utilities for a MIMO interference system as a global optimization problem with a rational objective function.
2) We apply the moment approach in [14], [15], [16] to obtain moment relaxations of the rational function optimization problem. Each moment relaxation is expressible as a linear SDP problem. Solving a sequence of such SDP relaxations gives rise to increasingly tight upper bounds on the global optimal objective value of the sum of sigmoid utilities.
3) In our simulation examples, we solve our first optimization model and the alternative model to determine the performance differences. Based on the results, we obtain insights about how interference and data-rate requirements can impede link-consumers' ability to maximize their sum of utilities.

[2] An arbitrary set $\mathbf{X} \subseteq \mathbb{R}_+^n$ satisfies the normality property if $[0,x] \subseteq \mathbf{X}$, $\forall x \in \mathbf{X}$.

C. Organization of Paper

The solution method in this paper is based on techniques in [14], [15], [16]. To aid understanding, we introduce relevant concepts in the remainder of this section. In II, we give the problem formulation and the algorithm for computing the global optimal sum of sigmoid utilities for a MIMO interference system. In III, we compute and compare the average global maxima sum of utilities achievable by our optimization model from II relative to the alternative model in four simulation examples. In IV, we conclude our findings.

D. Notations and Mathematical Preliminaries

- $A^\dagger$, A^T, α^T : respectively complex conjugate, matrix and vector transpose.
- $\mathrm{vec}(X)$: Gives the vectorized form of the upper triangular real and imaginary values in a Hermitian $X \in \mathbb{C}^{l\times l}$, $l \in \mathbb{N}$ where $X^\dagger = X$.
- $\mathbb{R}^d[x]$, $\deg(p(x))$: respectively the ring of real polynomials in $x \in \mathbb{R}^n$ of degree d or less, and degree of polynomial $p(x) \in \mathbb{R}[x]$.
- $\mathbf{Q}$: index set of all links.
- $x_q \in \mathbf{X}_q \triangleq \{x_q \in \mathbb{R}^{n_q} \mid 0 \le h_i(x_q) \in \mathbb{R}[x],\ i \in \mathscr{I}_q\}$, $q \in \mathbf{Q}$: set of vectorized inputs of link q defined by a set of constraints $\{0 \le h_i(x_q) \in \mathbb{R}[x]\}$ indexed by the set $\mathscr{I}_q$.
- $x \in \mathbf{X} \triangleq \prod_{q\in\mathbf{Q}} \mathbf{X}_q \subset \mathbb{R}^n$: set of joint vectorized inputs of all links in $\mathbf{Q}$ where $\sum_{q\in\mathbf{Q}} n_q = n$.
- $\mathbb{S}^l$: set of real symmetric matrices of size $l \in \mathbb{N}$.
- $|\alpha| \triangleq \sum_{i=1}^n \alpha_i$: sum of elements in $\alpha \in \mathbb{N}_0^n$.
- $\mathbf{S}_n^d \triangleq \{\alpha \in \mathbb{N}_0^n \mid d \ge |\alpha|\}$: set of degree vectors with sum up to d with cardinality $\mathrm{s}(n,d) \triangleq (n+d)!/(n!d!)$.
- $\mathscr{P}_\mathbf{X}$: set of all probability measures over $\mathbf{X}$.
- $\mathscr{M}_\mathbf{X} \supset \mathscr{P}_\mathbf{X}$: set of all finite Borel measures over $\mathbf{X}$.

Definition 1: (Monomial): Given $x \in \mathbb{R}^n$ and $\alpha \in \mathbb{N}_0^n$, the single term $x^\alpha \triangleq \prod_{i=1}^n x_i^{\alpha_i}$ is a monomial with degree $|\alpha|$.

Definition 2: (Monomial basis): Given $x \in \mathbb{R}^n$, the sequence $b^d(x) \triangleq (x^\alpha)_{\alpha\in\mathbf{S}_n^d}$ is a monomial basis in x with degree up to d that follows the negative degree lexicographic ordering[3].

Definition 3: (Moment): Given degree vector $\alpha \in \mathbf{S}_n^d$, $y_\alpha \triangleq \int_\mathbf{X} x^\alpha \,\mathrm{d}\pi(x) \in \mathbb{R}$ is the moment of order α for a measure π on $\mathbf{X}$, and $y \triangleq (y_\alpha)_{\alpha\in\mathbf{S}_n^d} \in \mathbb{R}^{\mathbf{S}_n^d}$ is the corresponding moment sequence.

Definition 4: (Moment matrix): Given $y \in \mathbb{R}^{\mathbf{S}_n^{2d}}$, $M^d(y) \triangleq \int_\mathbf{X} b^d(x) b^d(x)^T \mathrm{d}\pi(x) \in \mathbb{S}^{\mathrm{s}(n,d)}$ is the moment matrix of order d. Re-expressing in moments, the matrix entry indexed by (α,β) is $[M^d(y)]_{\alpha,\beta} = y_{\alpha+\beta}$ for $\alpha,\ \beta \in \mathbf{S}_n^d$.

Definition 5: (Localizing operator): Given $h(x) \in \mathbb{R}^{d'}[x]$ and an arbitrary moment sequence $y \in \mathbb{R}^{\mathbf{S}_n^d}$ where $d' \le d$, the localizing operator denoted by "$*$" for which $h * y \in \mathbb{R}^{\mathbf{S}_n^{d-d'}}$ is defined as $(h*y)_\alpha \triangleq \sum_{\gamma\in\mathbf{S}_n^{d'}} h_\gamma y_{\alpha+\gamma}$ for all $\alpha \in \mathbf{S}_n^{d-d'}$.

Definition 6: (Localizing matrix): Given $h(x) \in \mathbb{R}^{d'}[x]$ and an arbitrary moment sequence $y \in \mathbb{R}^{\mathbf{S}_n^{2d}}$ where $d' \le 2d$, $M^{d-\lceil d'/2\rceil}(h*y) \triangleq \int_\mathbf{X} b^d(x) b^d(x)^T h(x) \mathrm{d}\pi(x) \in \mathbb{S}^{\mathrm{s}(n,d-\lceil d'/2\rceil)}$ is the localizing matrix of order $d - \lceil d'/2\rceil$. Re-expressing in moments, the matrix entry indexed by (α,β) is $\left[M^{d-\lceil d'/2\rceil}(h*y)\right]_{\alpha,\beta} = \sum_{\gamma\in\mathbf{S}_n^{d'}} h_\gamma y_{\alpha+\beta+\gamma}$ for $\alpha,\ \beta \in \mathbf{S}_n^{d-\lceil d'/2\rceil}$.

[3]Given $x \in \mathbb{R}^n$, $\alpha,\beta \in \mathbb{N}_0^n$, the monomial ordering $x^\beta \succ_{\mathrm{negdeglex}} x^\alpha$ holds if and only if $|\beta| < |\alpha|$ or $|\beta| = |\alpha|$, $\exists i \in \{1,\ldots,n\}$ such that $\alpha_{i'} = \beta_{i'}$, $\forall i' \in \{1,\ldots,i-1\}$, $\alpha_i < \beta_i$.

II. Problem Formulation and Solution

In II-A, we derive a sum of utilities maximization problem formulation with positive semidefinite constraints for a MIMO interference system. In II-B, we re-express the sum of utilities maximization as a global optimization problem with polynomial constraints and a rational objective function. We then apply the moment approach in II-C to reformulate the global optimization problem in the moment form. In II-D, we arrive at a finite-dimensional moment relaxation, which is expressible as a linear SDP problem. In II-E, we specify a sequential SDP algorithm. In II-F, we discuss a convergence property of the algorithm.

A. Sum of Rational Utilities Maximization for a MIMO Interference System

We refer to the MIMO system capacity model in [10]. We denote the index set of the MIMO links by $\mathbf{Q}$, and introduce the domain sets of Hermitian covariance matrix inputs of the TX nodes as $\mathscr{X}_q \triangleq \{X_q \in \mathbb{C}^{n_{\mathrm{T}_q}\times n_{\mathrm{T}_q}} \mid X_q \succeq 0\}$[4], $\forall q \in \mathbf{Q}$ where n_{T_q} is the number of antennas at the TX node of link q, and $\mathscr{X} \triangleq \prod_{q\in\mathbf{Q}} \mathscr{X}_q$. We assume CSI is available at the TXs. We denote by $H_{q'q} \in \mathbb{C}^{n_{\mathrm{R}_q}\times n_{\mathrm{T}_{q'}}}$, q', $q \in \mathbf{Q}$ the complex Gaussian distributed MIMO channel matrix between the TX node in link q' and the RX node in link q, where n_{R_q} is the number of antennas at the RX node of link q.

We denote by R_q the total interference plus noise picked up by the RX node of link q. Assuming R_q is Gaussian distributed, it is expressible as

$$R_q \triangleq \sum_{q'\in\mathbf{Q}\setminus\{q\}} \rho_{q'q} H_{q'q} X_{q'} H_{q'q}^\dagger + I,\ \forall q \in \mathbf{Q}, \tag{1}$$

where I is the identity matrix, $X_{q'} \in \mathscr{X}_{q'}$, and $\rho_{q'q}$ is the interference-to-noise-ratio (INR) per unit transmit power with respect to the interference generated by the TX node in link q' but is picked up by the RX node in link q. We denote the mutual information of link q by $\lambda_q(X)$, $X \triangleq \prod_{q\in\mathbf{Q}} X_q \in \mathscr{X}$,

$$\lambda_q(X) \triangleq \log_2\left[\det(R_q + W_q)/\det(R_q)\right],\ \forall q \in \mathbf{Q}, \tag{2}$$

where $W_q = \rho_{qq} H_{qq} X_q H_{qq}^\dagger$, $\det(.)$ is matrix determinant, ρ_{qq} is the signal-to-noise-ratio (SNR) per unit transmit power with respect to the signal generated by the TX node of link q and is picked up by the RX node in the same link. We elaborate the modeling details of SNR and INR in III-A. As explained in Section I, we apply the sigmoid function on the mutual information to obtain the utility function of link q,

$$u_q(X) \triangleq 1/\left(1 + \tau^{\left[-a_q\lambda_q(X)+b_q\right]}\right),\ \forall q \in \mathbf{Q}, \tag{3}$$

where $a_q \in \mathbb{R}_{++}$, $b_q \in \mathbb{R}_{++}$, and $\tau \in \mathbb{R}_{++}$.

The data-rate needs of a link-consumer commensurate with the magnitude of b_q. For ease of modeling and demonstration purpose, we model consumers with low data-rate needs using an arbitrary $b_q = 3$, and consumers with higher data-rate needs

[4]The constraints $X_q \in \mathbb{C}^{n_{\mathrm{T}_q}\times n_{\mathrm{T}_q}}$ and $X_q \succeq 0$ imply $X_q = X_q^\dagger$.

using a relatively larger value such as $b_q = 6$. We let $\tau = e$ (i.e. base of natural logarithm) as in [12]. Rewriting $\lambda_q(X)$ of (2) in base e, (3) becomes

$$u_q(X) = G_q / \left(G_q + e^{b_q} [\det(R_q)]^{a_q \log_2 e} \right), \forall q \in \mathbf{Q}, \quad (4)$$

where $G_q = [\det(R_q + W_q)]^{a_q \log_2 e}$. In order for (4) to be a rational function, we assume a_q is chosen by design to satisfy $a_q \log_2 e \in \mathbb{N}$. Therefore the formulation for maximizing the sum of utilities given a specific realization of the channel matrices $H_{q'q} \in \mathbb{C}^{n_{\mathrm{R}_q} \times n_{\mathrm{T}_{q'}}}$, q', $q \in \mathbf{Q}$ is

$$\begin{aligned} \Phi_{\mathscr{X}} \ : \ \max_{X \in \mathscr{X}} \ & \sum_{q \in \mathbf{Q}} u_q(X) \\ \text{s.t.} \ & \left. \begin{array}{l} \mathrm{tr}(X_q) \le \mathrm{P}_{\max}, \\ X_q \succeq 0, \end{array} \right\} \forall q \in \mathbf{Q}, \end{aligned}$$

where tr(.) is matrix trace, and $\mathrm{P}_{\max}$ is the maximum power limit. Note problem $\Phi_{\mathscr{X}}$ is not a linear SDP due to its rational objective function.

B. Reformulation of $\Phi_{\mathscr{X}}$ as a Global Optimization Problem with Polynomial Constraints and a Rational Objective Function

Before applying the moment approach, we need to reformulate problem $\Phi_{\mathscr{X}}$ as a global optimization problem with polynomial constraints. To achieve this, we first vectorize the tuple of Hermitian covariance matrix inputs $X \in \mathscr{X}$ to obtain $x \in \mathbb{R}^{\sum_{q \in \mathbf{Q}} n_q}$ where $n_q = n_{\mathrm{T}_q}^2$ such that $x = \prod_{q \in \mathbf{Q}} \mathrm{vec}(X_q)$. The power limit constraints of problem $\Phi_{\mathscr{X}}$ are readily expressible as linear constraints in x. In order to replace the positive semidefinite constraints of problem $\Phi_{\mathscr{X}}$ with polynomial constraints, we introduce the following remark.

Remark 1: In problem $\Phi_{\mathscr{X}}$, each positive semidefinite constraint $X_q \succeq 0$, $q \in \mathbf{Q}$ is equivalent to enforcing non-negativity of every principal minor of X_q [24]. By representing X_q as a symbolic matrix, one can obtain principal minors of X_q using an algorithm for computing symbolic matrix determinant such as [25].

Invoking Remark 1 and vectorizing $X \in \mathscr{X}$ to obtain the vector input x, the positive semidefinite constraints $X_q \succeq 0$, $\forall q \in \mathbf{Q}$ and the power limit constraints $\mathrm{tr}(X_q) \le \mathrm{P}_{\max}$ in problem $\Phi_{\mathscr{X}}$ are re-expressible as polynomial constraints $0 \le h_i(x_q) \in \mathbb{R}[x_q]$, $\forall i \in \mathscr{I}_q$, $q \in \mathbf{Q}$. Denoting the feasible set of x by $\mathbf{X}$, we give a result concerning the latter.

Proposition 1: The feasible set $\mathbf{X}$ is real basic semialgebraic and compact.

Replacing the rational $u_q(X)$ in (4) with $u_q(x)$, and owing to Proposition 1, problem $\Phi_{\mathscr{X}}$ is re-expressible as a global optimization problem $\Phi_{\mathbf{X}}$ with a rational objective function over the real basic compact semialgebraic set $\mathbf{X}$.

C. Reformulating Global Optimization Problem in Moment Form

We use the method by Bugarin *et al.* in [16] to reformulate problem $\Phi_{\mathbf{X}}$ in the moment form. We first re-express $\Phi_{\mathbf{X}}$ as a sum of utilities maximization problem with respect to arbitrary probability measures $\pi \in \mathscr{P}_{\mathbf{X}}$. We denote the revised problem by Φ_π, in which we apply the integral operator on the sum of utilities objective in $\Phi_{\mathbf{X}}$ with respect to π acting as the new decision variable.

Proposition 2: The problem formulation for maximizing the expected sum of utilities is

$$\Phi_\pi : \sup_{\pi \in \mathscr{P}_{\mathbf{X}}} \sum_{q \in \mathbf{Q}} \int_{\mathbf{X}} u_q(x) \ \mathrm{d}\pi(x).$$

Remark 2: Since $u_q(x)$ in Proposition 2 is a rational function due to (4), the utility of link q is expressible as a ratio $u_q(x) = p_q(x)/g_q(x)$, where $p_q(x) \in \mathbb{R}[x]$, $g_q(x) \in \mathbb{R}[x]$. To be explained in Lemma 2, we have $g_q(x) > 0$, $\forall x \in \mathbf{X}$. To deal with the rational terms, we rewrite problem Φ_π in the following proposition.

Proposition 3: Introducing new measures $\pi_q \in \mathscr{M}_{\mathbf{X}}$, $\nu_q \in \mathscr{P}_{\mathbf{X}}$, $\forall q \in \mathbf{Q}$ such that for every $\mathbf{B} \subseteq \mathbf{X}$, we enforce $\int_{\mathbf{B}} \mathrm{d}\pi(x) = \nu_1(\mathbf{B}) \triangleq \int_{\mathbf{B}} g_1(x) \mathrm{d}\pi_1(x)$ and $\nu_q(\mathbf{B}) \triangleq \int_{\mathbf{B}} g_q(x) \mathrm{d}\pi_q(x) = \int_{\mathbf{B}} g_1(x) \mathrm{d}\pi_1(x)$, $\forall q \in \mathbf{Q} \backslash \{1\}$. Thus problem Φ_π is equivalent to the following with no rational term since the probability measures ν_q, $\forall q \in \mathbf{Q}$ are moment determinate (see subsection 4.4 in [26]).

$$\begin{aligned} \Phi_{\pi_{q \in \mathbf{Q}}} : \sup_{\pi_q \in \mathscr{M}_{\mathbf{X}}} & \sum_{q \in \mathbf{Q}} \int_{\mathbf{X}} p_q(x) \ \mathrm{d}\pi_q(x) \\ \text{s.t.} \ & \int_{\mathbf{X}} g_1(x) \ \mathrm{d}\pi_1(x) = 1, \\ & \int_{\mathbf{X}} x^\alpha g_q(x) \ \mathrm{d}\pi_q(x) = \int_{\mathbf{X}} x^\alpha g_1(x) \ \mathrm{d}\pi_1(x), \\ & \forall \alpha \in \mathbb{N}_0^n, \ q \in \mathbf{Q} \backslash \{1\}. \end{aligned}$$

To aid understanding, we first explain the definition of a quadratic module $\mathbf{M}((h_i)_{i \in \mathscr{I}}, \ 2d)$ given by the following,

$$\begin{aligned} \mathbf{M}((h_i)_{i \in \mathscr{I}}, \ 2d) \triangleq \{ & s_0(x) + h_i(x) s_i(x) \mid \\ & s_{i \in \{0\} \cup \mathscr{I}}(x) = \sum_{j \in \mathbf{J} \subset \mathbb{N}} f_j^2(x), \ f_j(x) \in \mathbb{R}[x], \\ & \deg(h_i(x) s_i(x)) \le 2d, \deg(s_0(x)) \le 2d \}, \end{aligned}$$

where $2d$ is the upper degree bound, and $0 \le h_i(x) \in \mathbb{R}[x]$, $i \in \mathscr{I}$ are the constraints defining an arbitrary real basic compact semialgebraic set. We henceforth assume the polynomial constraints $0 \le h_i(x_q) \in \mathbb{R}[x_q]$, $\forall i \in \mathscr{I}_q$, $q \in \mathbf{Q}$ generating the quadratic module $\mathbf{M}\left((h_i)_{i \in \mathscr{I}_q, q \in \mathbf{Q}}, \ 2d \right)$ are augmented with an additional bound constraint of the form $x^T x \le \omega$ with constant $\omega \in \mathbb{R}_{++}$. Such a bound constraint ensures that $\mathbf{M}\left((h_i)_{i \in \mathscr{I}_q, q \in \mathbf{Q}}, \ 2d \right)$ satisfies the Archimedean property [15], which enables one to invoke Theorem 4.17 of [15] to relax problem $\Phi_{\pi_{q \in \mathbf{Q}}}$ into a finite-dimensional moment relaxation problem.

D. Solving the Moment Optimization Problem as a Linear SDP

To solve $\Phi_{\pi_{q \in \mathbf{Q}}}$, we relax the problem using truncated moment sequences $y_q \triangleq (y_{q\gamma})_{\gamma \in \mathbf{S}_n^{2d}}$, $q \in \mathbf{Q}$. Therefore, we arrive at the following moment relaxation of order $d \in \mathbb{N}$, where

$2d \geq \deg\left(\sum_{q\in\mathbf{Q}} p_q(x)\right)$.

$$\begin{aligned}
\Phi^d_{y_{q\in\mathbf{Q}}}: \max_{y_q\in\mathbb{R}^{\mathbf{S}^{2d}_n}} \quad & \sum_{q\in\mathbf{Q}} {p_q}^T y_q \\
\text{s.t.} \quad & \left.\begin{array}{l} M^d_q(y_q) \succeq 0, \\ M^{d-d_i}_q(h_i * y_q) \succeq 0,\ \forall i\in\mathscr{I}_q, \end{array}\right\}\forall q\in\mathbf{Q}, \\
& \sum_{\beta\in\mathbf{S}^{d_{g_q}}_n} g_{q,\beta} y_{q,\beta} = 1,\ q=1, \\
& \sum_{\beta\in\mathbf{S}^{d_{g_q}}_n} g_{q,\beta} y_{q,\alpha+\beta} = \sum_{\beta\in\mathbf{S}^{d_{g_1}}_n} g_{1,\beta} y_{1,\alpha+\beta}\ , \\
& \quad \forall\alpha\in\mathbf{S}^{2d-\max(d_{g_q},d_{g_1})}_n,\ q\in\mathbf{Q}\backslash\{1\},
\end{aligned} \tag{5}$$

where $M^d_q(y_q)\in\mathbb{S}^{s(n,d)}$, $M^{d-d_i}_q(h_i*y_q)\in\mathbb{S}^{s(n,d-d_i)}$ are respectively the moment and localizing matrices, $p_q(x)$, $g_q(x)$ from $\Phi_{\pi_{q\in\mathbf{Q}}}$, and d_i, $d_{g_q}\in\mathbb{N}$ are defined respectively as $d_i \triangleq \lceil\deg(h_i(x))/2\rceil$ and $d_{g_q}\triangleq\deg(g_q(x))$. In problem $\Phi^d_{y_{q\in\mathbf{Q}}}$, the objective and the equalities are affine in $y_{q\in\mathbf{Q}}$, and the non-linear constraints are positive semidefinite in $y_{q\in\mathbf{Q}}$. Therefore problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ is solvable as a linear SDP problem. Using a sufficiently large relaxation order d, one can use Gloptipoly 3 [17] to solve $\Phi^d_{y_{q\in\mathbf{Q}}}$ for the global solutions subject to moment matrix rank condition in Theorem 6.18 of [15].

E. Sequential SDP Algorithm

We highlight the steps for solving problem $\Phi^d_{y_{q\in\mathbf{Q}}}$. We let the iteration counter be $k\in\mathbb{N}$, and let $d_k\in\mathbb{N}$ be the moment relaxation order at iteration k.

1) Select a tolerance $\xi>0$, set iteration counter $k=1$, $d_1=\max_{q\in\mathbf{Q}}\lceil\deg(u_q(x))/2\rceil$ and $z_{k-1}=+\infty$.
2) Solve problem $\Phi^{d_k}_{y_{q\in\mathbf{Q}}}$ with an SDP tool such as [17], denote the optimal objective value by z^*_k, and the optimal moment sequence by y^*_k.
3) If the moment matrix rank condition (see II-D) is satisfied or $z^*_{k-1}<+\infty$ and $|(z^*_k-z^*_{k-1})/z^*_{k-1}|\leq\xi$ hold true, terminate and return $(z^*_k,\ y^*_k)$. Otherwise, set $d_{k+1}=d_k+1$, increment k and repeat Step 2.

F. Convergence Property of Sequential SDP Algorithm

In this subsection we assume the number of global maximizing solutions to problem $\Phi_{\pi_{q\in\mathbf{Q}}}$ is finite, and use results in [27] to give a convergence property of the algorithm in II-E.

Lemma 1: If the number of global solutions to $\Phi_{\pi_{q\in\mathbf{Q}}}$ is finite, then there exists a constant $\tau\in\mathbb{N}$ such that $\alpha\in\mathbb{N}^n_0$ in problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ satisfies $|\alpha|\leq 2\min(d,\tau)-\max_{q\in\mathbf{Q}} d_{g_q}$.

Since strong duality holds [14], we work with the dual formulation of problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ in (5) through the following lemmas to give a theorem describing convergence of the dual objective values. To be consistent with [27], we assume the input domain $\mathbf{X}$ is affine-transformed into $(-1,1)^n$. Following from II-C, we assume $\mathbf{M}\left((h_i)_{i\in\mathscr{I}_q,q\in\mathbf{Q}},\ 2d\right)$ satisfies the Archimedean property [15], which is a pre-requisite for invoking results in [27].

Lemma 2: In Remark 2, the numerator and denominator of $u_q(x)$ namely $p_q(x)$ and $g_q(x)$, $\forall q\in\mathbf{Q}$ respectively satisfy $p_q(x)>0$ and $g_q(x)>0$, $\forall x\in\mathbf{X}$.

For convenience, we henceforth introduce $\mathbf{T}^d\triangleq(\mathbf{Q}\backslash\{1\})\times\mathbf{S}^{2\min(d,\tau)-\max_{q\in\mathbf{Q}} d_{g_q}}_n$ where τ from Lemma 1.

Lemma 3: Assuming there is a finite number of global maximizing solutions, the dual formulation of problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ in (5) is

$$\begin{aligned}
&\min\ \lambda_1 \\
&\text{s.t. } 0\leq\lambda_1\in\mathbb{R},\ \left(\lambda_{q\gamma}\right)_{(q,\gamma)\in\mathbf{T}^d}\in\mathbb{R}^{|\mathbf{T}^d|}, \\
&\quad -\sum_{q\in\mathbf{Q}} p_q(x) \\
&\quad -\sum_{q\in\mathbf{Q}\backslash\{1\}}\sum_{\alpha\in\mathbf{S}^{m_1}_n}\sum_{\gamma\in\mathbf{S}^{m_1-m_2}_n,\beta\in\mathbf{S}^{m_2}_n|\gamma+\beta=\alpha}\lambda_{q\gamma}(-g_{1,\beta}+g_{q,\beta})x^\alpha \\
&\quad +\lambda_1 g_1(x)\in\mathbf{M}\left((h_i)_{i\in\mathscr{I}_q,q\in\mathbf{Q}},\ 2d\right),
\end{aligned} \tag{6}$$

where $\mathbf{M}\left((h_i)_{i\in\mathscr{I}_q,q\in\mathbf{Q}},\ 2d\right)$ is the quadratic module, $m_1=2\min(d,\tau)$, $m_2=\max(d_{g_q},d_{g_1})$, and τ from Lemma 1.
For notational convenience, we henceforth denote the quadratic module membership constraint in problem (6) by

$$\lambda_{1,d}g_1(x)-p\left(\left(\lambda_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ x\right)\in\mathbf{M}\left((h_i)_{i\in\mathscr{I}_q,q\in\mathbf{Q}},\ 2d\right), \tag{7}$$

where $\left(\lambda_{1,d},\left(\lambda_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d}\right)$ is a feasible solution to (6) at relaxation order d. Due to the definition of the quadratic module, the left-hand-side of (7) is non-negative over $\mathbf{X}$.

Lemma 4: We denote the optimal solution tuple obtained from solving problem (6) of Lemma 3 at relaxation order d by $\left(\lambda^*_{1,d},\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d}\right)$. Let $\hat{x}_d=\arg\min_{x\in\mathbf{X}}\lambda^*_{1,d}g_1(x)-p\left(\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ x\right)$. We then have

$$\begin{aligned}
&\lambda^*_{1,d}g_1(\hat{x}_d)-p\left(\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ \hat{x}_d\right)\leq \\
&\quad f(d)\left\|\lambda^*_{1,d}g_1(x)-p\left(\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ x\right)\right\|,
\end{aligned}$$

where $f(d)\triangleq\frac{6\tau^3n^{2\tau}}{\sqrt[c]{\log(2d/c)}}$, τ from Lemma 1, c is a constant dependent on $(h_i(x))_{i\in\mathscr{I}_q,q\in\mathbf{Q}}$ (see [27]), and norm $\|.\|$ is defined as $\|p(x)\|\triangleq\max_\alpha\frac{|p_\alpha|(\alpha_1!\cdots\alpha_n!)}{(|\alpha|!)}$ (see [27]) for arbitrary $p(x)\in\mathbb{R}[x]$.

To describe complexity of algorithm II-E, we give an upper bound on $\lambda^*_{1,d}-\lambda^*_{1,\infty}$ for an arbitrary relaxation order d since $\lambda^*_{1,\infty}$ is the desired optimal dual objective value. We let $\underline{d}$ be the minimum relaxation order satisfying $2\underline{d}\geq\max(\deg(\sum_{q\in\mathbf{Q}}p_q(x)),\ \max_{q\in\mathbf{Q}}d_{g_q},\ 2\max_{i\in\mathscr{I}_q,q\in\mathbf{Q}}d_i)$. For arbitrary $\mathbf{D}\subseteq\mathbb{N}$, we define $\Lambda_\mathbf{D}\triangleq\left\{\left(\lambda_{1,d},(\lambda_{q\gamma,d})_{(q,\gamma)\in\mathbf{T}^d}\right)\in\mathbb{R}_+\times\mathbb{R}^{|\mathbf{T}^d|}|d\in\mathbf{D}\right\}$, denote by $\Lambda^*_\mathbf{D}$ the set containing all optimal solution tuples $\left(\lambda^*_{1,d},\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d}\right)$ obtained from solving (6) at relaxation order $d\in\mathbf{D}$, and define $\mathrm{n}(\Lambda_\mathbf{D})\triangleq\sup_{(\lambda_1,\lambda_{q\gamma})\in\Lambda_\mathbf{D}}\left\|\lambda_1g_1(x)-p\left(\lambda_{q\gamma},\ x\right)\right\|$.

Theorem 1: Consider the dual relaxation problem in (6) at order d. The expression $\lambda^*_{1,d}-\lambda^*_{1,\infty}$ is upper bounded by

$$\begin{aligned}
\lambda^*_{1,d}-\lambda^*_{1,\infty}\leq&\left(1/\min_{x\in\mathbf{X}}g_1(x)\right)\Big[f(d)\mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,\infty\}}) \\
&-\left(\lambda^*_{1,\infty}g_1(\hat{x}_d)-p\left(\left(\lambda^*_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ \hat{x}_d\right)\right)\Big],
\end{aligned} \tag{8}$$

where $\underline{d}$ is the minimum relaxation order, $\hat{x}_d$ and $f(d)$ from Lemma 4, and $\lambda_{1,d}^* - \lambda_{1,\infty}^* \to 0$ as $d \to \infty$.

Proof: Let the optimal solution tuple obtained from solving problem (6) at infinite relaxation order be $\left(\lambda_{1,\infty}^*, \left(\lambda_{q\gamma,\infty}^*\right)_{(q,\gamma)\in\mathbf{T}^\infty}\right)$ satisfying

$$\lambda_{1,\infty}^* g_1(x) - p\left(\left(\lambda_{q\gamma,\infty}^*\right)_{(q,\gamma)\in\mathbf{T}^\infty},\ x\right) \in \mathbf{M}\left((h_i)_{i\in\mathscr{I}_q, q\in\mathbf{Q}},\ \infty\right). \tag{9}$$

Due to [16], the primal problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ in (5) is feasible and bounded. The dual problem in (6) is feasible because we can construct a feasible solution by setting $\left(\lambda_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d} = \{0\}^{|\mathbf{T}^d|}$ and $\lambda_{1,d}$ to a sufficiently large value such that the positive term $\lambda_{1,d} g_1(x)$ (see Lemma 2) becomes arbitrarily large hence satisfying the constraint $\lambda_{1,d} g_1(x) - p\left(\left(\lambda_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d},\ x\right) \in \mathbf{M}\left((h_i)_{i\in\mathscr{I}_q, q\in\mathbf{Q}},\ 2d\right)$. Since problem (5) is feasible and bounded, the dual values $\left(\lambda_{1,d}, \left(\lambda_{q\gamma,d}\right)_{(q,\gamma)\in\mathbf{T}^d}\right)$ are bounded thus implying the left-hand-side of (9) is also bounded. Therefore $\Lambda^*_{\{\underline{d},\ldots,\infty\}}$ is a bounded set. Since the property $\mathbf{M}\left((h_i)_{i\in\mathscr{I}_q, q\in\mathbf{Q}},\ 2d\right) \subset \mathbf{M}\left((h_i)_{i\in\mathscr{I}_q, q\in\mathbf{Q}},\ \infty\right)$ results in $\mathrm{n}(\Lambda^*_{\{d\}}) \le \mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,d\}}) \le \mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,\infty\}})$, and due to Lemma 4, we have $\lambda_{1,d}^* g_1(\hat{x}_d) - p\left(\left(\lambda_{q\gamma,d}^*\right)_{(q,\gamma)\in\mathbf{T}^d},\ \hat{x}_d\right) \le f(d)\mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,\infty\}})$. Since $f(d)\mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,\infty\}})$ is finite positive, a finite upper bound on $\lambda_{1,d}^* - \lambda_{1,\infty}^*$ is given by

$$\lambda_{1,d}^* - \lambda_{1,\infty}^* \le (1/g_1(\hat{x}_d))\Big[f(d)\mathrm{n}(\Lambda^*_{\{\underline{d},\ldots,\infty\}}) - \left(\lambda_{1,\infty}^* g_1(\hat{x}_d) - p\left(\left(\lambda_{q\gamma,d}^*\right)_{(q,\gamma)\in\mathbf{T}^d},\ \hat{x}_d\right)\right)\Big].$$

Consider the following expression in (8),

$$\lambda_{1,\infty}^* g_1(\hat{x}_d) - p\left(\left(\lambda_{q\gamma,d}^*\right)_{(q,\gamma)\in\mathbf{T}^d},\ \hat{x}_d\right). \tag{10}$$

Due to Lemma 4, both (10) and $f(d)$ in (8) approach zero as $d \to \infty$. ■

III. Wireless Application

In III-A, we describe the MIMO interference system. In III-B, we specify the system parameters. In III-C we illustrate the results. To compute the average global maxima sum of utilities achievable by our optimization model (see problem $\Phi^d_{y_{q\in\mathbf{Q}}}$) and by the alternative model that optimizes only the eigenvalues, we generate 1000 scenarios with random channel matrices $H_{q'q}$. For each scenario, we solve our optimization model in problem $\Phi^d_{y_{q\in\mathbf{Q}}}$ up to an appropriate moment relaxation order d, based on the steps in II-E. In III-D, we tabulate the performance differences between our covariance matrices optimization model and the alternative eigenvalues optimization model.

A. MIMO Interference System Model

We use the flat Rayleigh fading narrowband MIMO channels with log-normal shadowing in [7]. Let the index set of the MIMO links be $\mathbf{Q}$. We denote the maximum distance between a TX node and the corresponding RX node in any link by $\mathrm{d_{max}}$ in meters, and the minimum separating distance between any two nodes by $\mathrm{d_{min}}$. We set up the TX and RX nodes to reside in box areas of varying lengths that influence the interference intensity among the $|\mathbf{Q}|$-links. The SNR ρ_{qq} and INR $\rho_{q'q}$ in (1) are defined by $\rho_{q'q} \triangleq \left(d_{\max}/d_{q'q}\right)^\gamma 10^{(\mathrm{SNR_{min}}-s)/10}$, $\forall q',\ q \in \mathbf{Q}$, where $\mathrm{SNR_{min}}$ dB (decibel) is required for successful transmission, $d_{q'q}$ is the distance between the TX node in link q' and the RX node in link q, γ is the path loss exponent, and log-normal shadowing random variable s has zero mean and standard deviation σ_s dB. For a MIMO channel, the Kronecker model [28], [29] assumes that the spatial correlation of the antennas at a TX node is independent and separable from the antennas' correlation at other nodes. To model antennas' correlation, we adopt the correlation model in [30]. We quantify antennas' correlation at a RX node q with c_{R_q}, and antennas' correlation at a TX node q' with $c_{\mathrm{T}_{q'}}$.

TABLE I
PERCENTAGES OF SCENARIOS VERSUS REQUIRED RELAXATION ORDERS

Percentage of Scenarios	Required Relaxation Orders
90.19 %	3 or less
6.37 %	4
3.44 %	Fail to converge.

B. Parameter Settings

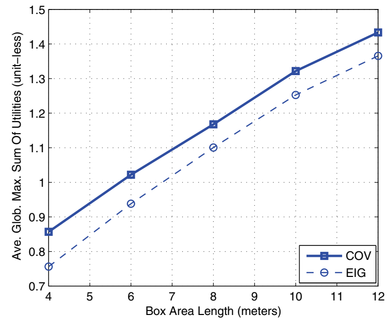
We set $n_{\mathrm{T}_q} = n_{\mathrm{R}_q} = 2$, $\forall q \in \mathbf{Q}$, $\mathrm{d_{min}} = 1$ m, $\mathrm{d_{max}} = 2$ m. For log-normal shadowing, we set $\sigma_s = 1$ dB, $\gamma = 3$. Assuming strong antennas' correlation, we set $c_{\mathrm{R}_q} = c_{\mathrm{T}_q} = 0.9$, $\forall q \in \mathbf{Q}$. We set $\mathrm{P_{max}} = 1$ Watt, $\mathrm{SNR_{min}} = 20$ dB, and define the sigmoid utility function in (4) using $a_q = \frac{1}{\log_2 e}$, $\forall q \in \mathbf{Q}$. In each example, we vary the box area length to influence interference, and respectively use $b_q = 3$ and 6 to model consumers with low and high data-rate needs.

C. Simulation Results

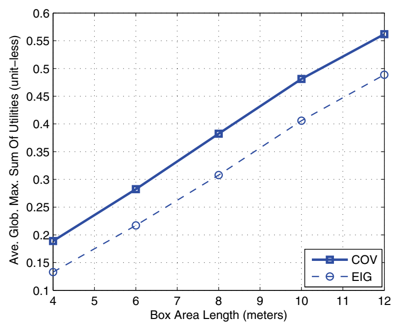
Recall in II-B that our model optimizes *covariance matrices* of the TXs, in contrast to the alternative model that optimizes only the *eigenvalues*. With this in mind, we refer to our optimization model by "COV" and to the alternative model by "EIG" in the following results depicted in Figures 1(a)-1(d). In the simulation results, an increase in b_q (i.e., consumers switch from low data-rate needs to high data-rate needs) causes the average global maximum sum of utilities to deteriorate. In table I, we tabulate the different percentages of all generated scenarios that require varying numbers of relaxation orders in order to achieve convergence. Interestingly, although the convergence bound in Theorem 1 is conservative, the algorithm in practice converges in as much as 95% of the generated scenarios according to Table I. For those scenarios where the algorithm fails to converge, it is because larger relaxation orders such as 5 or higher result in large SDP relaxations that cause the solver to consume excessive time and hardware memory resources.

D. Under-Performance Due To Interference And Excessive Data-Rate Requirement

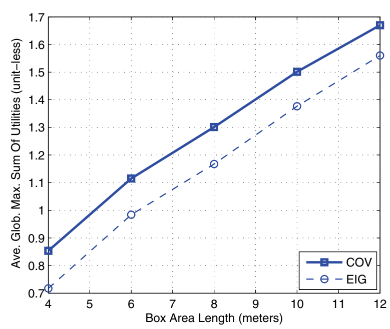
Table II shows the differences in performance using our "COV" model over the "EIG" model. For example when link-



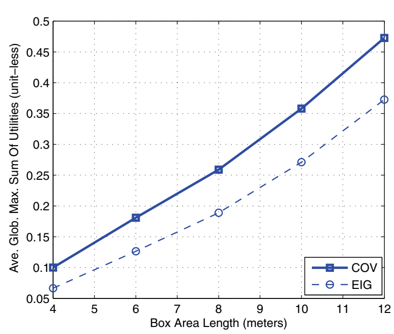
(a) 2 Low Data-Rate Consumers ($b_q = 3, q \in \{1,2\}$)



(b) 2 High Data-Rate Consumers ($b_q = 6,\ q \in \{1,2\}$)



(c) 3 Low Data-Rate Consumers ($b_q = 3,\ q \in \{1,2,3\}$)



(d) 3 High Data-Rate Consumers ($b_q = 6,\ q \in \{1,2,3\}$)

Fig. 1. Average Global Maxima Sum Of Utilities For Networks Of 2 and 3 Link-Consumers.

interference is strong and the application requires high data-rates, the two setups involving 2 and 3 links with box length=4 and $b_q = 6$ show that the "COV" model achieves 40%-50%

TABLE II
PERCENTAGE INCREASES IN AVERAGE GLOBAL MAXIMA SUM OF UTILITIES USING "COV" OVER "EIG" MODEL

Box Area Length	4	6	8	10	12
% Increase (2 links, $b_q = 3$)	13.34	8.93	6.10	5.52	4.96
% Increase (2 links, $b_q = 6$)	42.32	30.12	24.27	18.54	14.99
% Increase (3 links, $b_q = 3$)	18.96	13.32	11.46	9.07	7.03
% Increase (3 links, $b_q = 6$)	50.73	43.09	37.10	32.04	26.84

increase, meaning it becomes significantly difficult for the link-consumers to achieve global maxima sum of utilities when the MIMO system is subject to strong interference, and concurrently has to fulfill intensive application-specific data-rate requirements.

Although the examples focus on applications with inelastic network traffic where consumer utilities are modeled as sigmoid functions, insights can still be drawn to improve future MIMO system design work. The observations in Table II suggest that the designer should first evaluate whether a given system has sufficient capacity to satisfy the data-rate requirement of the application. The designer is advised to take mitigating steps to manage potential system interference only after the system has passed the minimum data-rate requirement. Such an approach enables link-consumers using non-global optimal but practical interference-avoidance algorithms to operate closer to the global maxima sum of utilities.

IV. CONCLUSION

Our demonstrations in subsections III-C and III-D show that solving for the instantaneous global maximum sum of sigmoid utilities as a global optimization problem with a rational objective function via the moment approach is feasible. Through optimizing the "COV" and "EIG" models, the simulation results demonstrate the negative consequences from coercing a MIMO system subject to interference into supporting applications with excessive data-rate requirements. The optimization method presented in this paper however relies on the scalability of SDP solvers. Standard solvers such as [31] based on the classical primal-dual interior point method [32] do not readily scale up to handle larger size SDP problems, thereby hindering the moment approach outlined in II from handling larger networks. For future work, we propose exploring large-scale SDP solvers such as the projection method in [33], [34].

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