491 Knowledge Representation

Tutorial Exercise

Logic databases

1. Let $p(x)$, $m(x)$, $f(x)$, $h(x, n)$ represent 'x is a person', 'x is male', 'x is female', 'the home telephone number of x is n'.

   Consider databases $\mathrm{Th}(D_i)$ for the following examples:

$$D_1 = \{p(a), p(b), m(a)\}$$
$$D_2 = \{p(a), p(b), \forall x\,(\,p(x) \to (m(x) \lor f(x))\,)\,\}$$
$$D_3 = \{p(a), p(b), m(a), m(b)\}$$
$$D_4 = \{\}$$

   and integrity contraint I1: 'every person is either male or female'.

   First, decide which of the databases $\mathrm{Th}(D_i)$ intuitively satisfy the integrity constraint I1. Then try the three definitions of integrity constraint satisfaction (consistency, entailment/theoremhood, metalevel/epistemic) and see what you get for each.

   Repeat the above for the databases $\mathrm{Th}(D_i)$:

$$D_5 = \{p(a), p(b), h(a, 123)\}$$
$$D_6 = \{p(a), p(b), \forall x\,(\,p(x) \to \exists n\, h(x, n)\,)\,\}$$
$$D_7 = \{p(a), p(b), \forall x\,(p(x) \to h(x, 456))\}$$

   and integrity contraint I2: 'every person has a home telephone number'.

   Finally, check again but this time on the databases

$$cwa_{\mathcal{P}}(D_i) = \mathrm{Th}(D_i \cup \{\neg\alpha \mid \alpha \in \mathcal{P},\ \alpha \notin \mathrm{Th}(D_i)\}\,)$$

   where $\mathcal{P} = \{p(a), p(b), m(a), m(b), f(a), f(b)\}$ for databases D_1–D_4, and
   $\mathcal{P} = \{p(a), p(b), h(a, 123), h(a, 456), h(b, 123), h(b, 456)\}$ for D_5–D_7.

2. (More demanding, but instructive.) Check the claim about relative strengths of integrity constraint satisfaction summarised in the lecture notes.

   (i) First check that any Cn (with $A \subseteq \mathrm{Cn}(A)$) satisfies (for all A, X, Y):

$$\text{If } X \vdash_{PL} Y \text{ then } A \vdash X \text{ implies } A \vdash Y$$
$$Y \subseteq \mathrm{Th}(X) \;\Rightarrow\; (X \subseteq \mathrm{Cn}(A) \;\Rightarrow\; Y \subseteq \mathrm{Cn}(A))$$

   iff Cn is 'closed under truth-functional consequence': $\mathrm{Th}(\mathrm{Cn}(A)) \subseteq \mathrm{Cn}(A)$. (The latter is a very reasonable property.)

   (ii) Now check that, for all α and β, if $(\alpha \to \beta) \in \mathrm{Cn}(D)$ then $\alpha \in \mathrm{Cn}(D)$ implies $\beta \in \mathrm{Cn}(D)$, and further, that as long as $\mathrm{Cn}(D)$ is consistent, if $\alpha \in \mathrm{Cn}(D)$ implies $\beta \in \mathrm{Cn}(D)$ then $\neg(\alpha \to \beta) \notin \mathrm{Cn}(D)$.

   (iii) Now the other direction, if $\mathrm{Cn}(D)$ is *complete* then, for all α and β: $\neg(\alpha \to \beta) \notin \mathrm{Cn}(D)$ implies if $\alpha \in \mathrm{Cn}(D)$ then $\beta \in \mathrm{Cn}(D)$, and further, if $\alpha \in \mathrm{Cn}(D)$ implies $\beta \in \mathrm{Cn}(D)$ then $(\alpha \to \beta) \in \mathrm{Cn}(D)$.

   How do these observations apply to the databases of Question 1?