491 Knowledge Representation

Tutorial Exercise

Fixpoint semantics for definite programs

Question 1

(a) Let $IDB = \{\, p \leftarrow q, r\, ;\ q \leftarrow s \,\}$ and $EDB = \{\, r,\ s \,\}$.
Check that $Cl_{IDB}(EDB) = T'_{IDB}\uparrow^{\omega}(EDB) \neq T_{IDB}\uparrow^{\omega}(EDB)$.

(b) Same again but for $IDB = \{\, p \leftarrow q, r\, ;\ q \leftarrow s\, ;\ r \,\}$ and $EDB = \{\, s \,\}$.

Question 2 (Optional) Following on from the previous question. Let P be a set of definite clauses and X any set of atoms of P.

(a) Show (by induction on n) that for all $n \geq 0$:

$$T_P\uparrow^n(X) \subseteq T'_P\uparrow^n(X)$$

and hence that $T_P\uparrow^{\omega}(X) \subseteq T'_P\uparrow^{\omega}(X)$.

(b) We know (e.g., from the example in Question 1) that in general $T_P\uparrow^{\omega}(X) \neq T'_P\uparrow^{\omega}(X)$.
But show (by induction on n) that for all $n \geq 0$, $T_P\uparrow^n(\emptyset) \subseteq T_P\uparrow^{n+1}(\emptyset)$, and hence

$$T'_P\uparrow^n(\emptyset) = T_P\uparrow^n(\emptyset)$$

from which follows $T_P\uparrow^{\omega}(\emptyset) = T'_P\uparrow^{\omega}(\emptyset)$.

Question 3

Suppose IDB is a set of definite clauses and EDB is a set of atoms. Show that $Cl_{IDB}(EDB)$ is the least Herbrand model of $IDB \cup EDB$.
Hint: if EDB is a set of atoms, then $T_{EDB}(X) = EDB$. And clearly $T_{P_1 \cup P_2}(X) = T_{P_1}(X) \cup T_{P_2}(X)$.

Question 4

Cl_{IDB} maps sets of atoms to sets of atoms. Show that if IDB is *definite* then Cl_{IDB} is a classical consequence operator (Tarski):

- $X \subseteq Cl_{IDB}(X)$
- $Cl_{IDB}(Cl_{IDB}(X)) \subseteq Cl_{IDB}(X)$
- if $X_1 \subseteq X_2$ then $Cl_{IDB}(X_1) \subseteq Cl_{IDB}(X_2)$

Question 5 Consider the following set of (normal, not definite) clauses IDB:

$$p \leftarrow r,\ \text{not}\ q$$
$$q \leftarrow r,\ \text{not}\ p$$

What is $Cl_{IDB}(\{r\})$? (Hint: there isn't one.)