Exercises

Program Analysis (CO70020)

Sheet 5

Exercise 1 *Consider the following imperative language with statements of the form:*

$$S \quad ::= \quad \texttt{x := a} \;\Big|\; \textbf{skip} \;\Big|\; S_1 \,;\, S_2 \;\Big|\; \textbf{if } b \textbf{ then } S_1 \textbf{ else } S_2 \;\Big|\; \textbf{while } b \textbf{ do } S$$
$$\Big|\; \textbf{choose } S_1 \mid S_2 \mid \ldots \mid S_n \;\Big|\; \textbf{combine } S_1 \mid S_2 \mid \ldots \mid S_n$$

In the **choose** *statement only one of the* $n \geq 1$ *statements* S_i *is actually selected to be executed. The* **combine** *executes all of the* n *statements* S_i *in some sequence. In both statements the choices are made non-deterministicly.*

Define a Live Variable Analysis LV*, similar to the one for the simple* **while** *language, for this extended language. Define an appropriate labelling for statements/blocks and give a definition for the flow flow (together with init and final).*

Exercise 2 *Consider the following expression from which labels have been stripped:*

$$\begin{array}{l}(\texttt{let } g = (\texttt{fn } f \texttt{ => } (\texttt{if } (f\ 3) \texttt{ then } 10 \texttt{ else } 5)) \\ \;\texttt{in } (g\ (\texttt{fn } y \texttt{ =>} (y > 2))\ )\ )\end{array}$$

Label the expression and give a brief and informal description of its execution: what does it evaluate to?

Write down the constraints for a 0-CFA and provide the least solution that satisfies the constraints.

Exercise 3 *Consider the following extraction function for* $n \in \mathbb{N}$*:*

$$\beta(n) = \begin{cases} \min \textit{ bits to represent } n & \textit{if } n < 2^8 \\ \textbf{overflow} & \textit{otherwise} \end{cases}$$

which allows for a Bit-Size analysis for "small" integers via Abstract Interpretation.

Describe the (abstract) property lattice and the concrete and abstract domain (incl. ordering and least upper bound operation). Furthermore, define the abstraction, α*, and concretisation,* γ*, functions.*

Construct formally the abstraction (in the sense of Abstract Interpretation) of the doubling and square function, i.e. $f^{\#}$ *and* $g^{\#}$ *for*

$$f(n) = 2 \times n \quad \textit{and} \quad g(n) = n^2$$

Exercise 4 *Consider a Sign Analysis for the imperative* WHILE *language. That is: We are interested in the* **sign** *of variables, i.e. whether we can guarantee that for a given program point and a variable x (at least) one of the following properties holds: $x = 0$, $x < 0$, $x > 0$, $x \leq 0$ and $x \geq 0$.*

Define a representation function β for this Sign Analysis. How can one define the corresponding correctness relation R_β? State formally what it means that the transfer functions f_ℓ for all labels are fulfilling the correctness condition.