Computing Minimal Deformations: Application to Construction of Statistical Shape Models

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Abstract

Nonlinear registration is mostly performed after initialization by a global, linear transformation (in this work, we focus on similarity transformations), computed by a linear registration method. For the further processing of the results, it is mostly assumed that this preregistration step completely removes the respective linear transformation. However, we show that in deformable settings, this is not the case. As a consequence, a significant linear component is still existent in the deformation computed by the nonlinear registration algorithm. For construction of statistical shape models (SSM) from deformations, this is an unwanted property: SSMs should not contain similarity transformations, since these do not capture information about shape. We propose a method which performs an a posteriori extraction of a similarity transformation from a given nonlinear deformation field, and we use the processed fields as input for SSM construction. For computation of minimal displacements, a closed-form solution minimizing the squared Euclidean norm of the displacement field subject to similarity parameters is used. Experiments on real inter-subject data and on a synthetic example show that the theoretically justified removal of the similarity component by the proposed method has a large influence on the shape model and significantly improves the results.

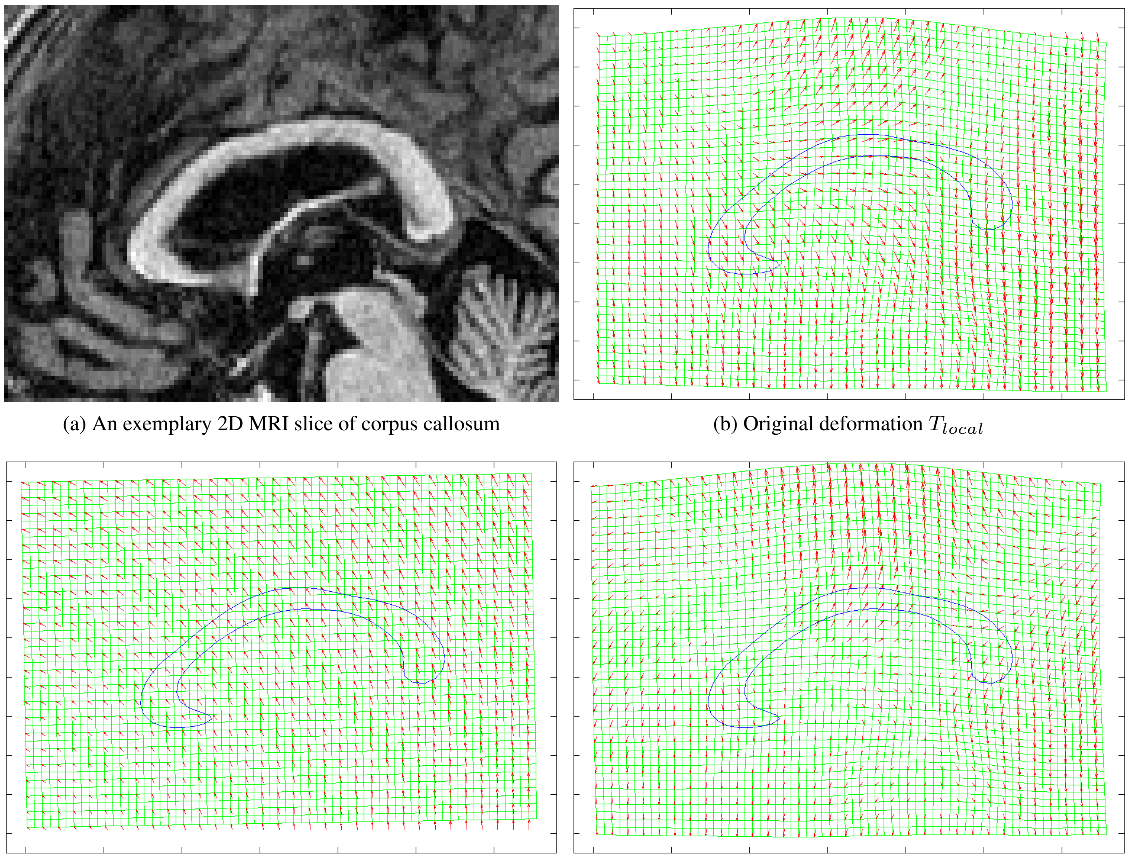
1. Introduction

This paper has two major goals. The first consists of pointing out that deformation fields resulting from standard registration schemes often contain a significant amount of linear transformation, and proposing a method to extract this linear component, thus computing *minimal nonlinear deformations*[1]. For a visualization of the setting, please refer to Fig. 1. Secondly, we identify the construction of shape models to be an application for which from the theoretical point of view, no similarity transformation should be contained in the deformations which are used to construct the model. If the similarity component is not extracted from the deformation, the first modes of the constructed model may not describe the largest variation in shape of the given samples. We show that by the proposed method, this negative effect can be eliminated, thus resulting in improved shape models.

In the following, we hope to provide an intuitive understanding why standard registration methods in general do not compute minimal deformations, and also why this is an important point when building shape models.

Nonlinear Registration and Resulting Deformations. Nonlinear registration is a technique which has been studied heavily over the last two decades. The goal of nonlinear registration is to estimate the nonlinear transformation which relates two given images. This can also be seen as computing the dense point correspondences between the two images. Virtually all schemes for nonlinear registration proposed in the literature employ a global linear preregistration step, followed by a nonlinear method accounting for local differences between the images. We make the observation that global registration methods cannot fully recover linear transformations in deformable scenarios. As a

[1] In this paper we distinguish between linear and nonlinear transformations. By *linear*, we constrain ourselves in the following mostly to the similarity transformation model. By *nonlinear* we understand any higher-order transformation and we refer to these also as *deformations*.



(a) An exemplary 2D MRI slice of corpus callosum

(b) Original deformation T_{local}

(c) Extracted similarity transformation T_{lin}

(d) Remaining minimal nonlinear transformation T_{nl}

Figure 1: Illustration of the discussed setting. On a set of images of corpus callosum **(a)** (see also Section 3.1.2), nonlinear registration is performed, resulting in a deformation field T_{local}, see **(b)**. A similarity component T_{lin} **(c)** is extracted from the original field T_{local}, resulting in a minimal nonlinear transformation T_{nl} **(d)**.

consequence, the deformation field which is computed by the local nonlinear registration algorithm contains a linear transformation. This occurs with intensity-based, as well as with landmark-based linear registration approaches. The reason for this is that for the exact estimation of the linear part, all point correspondences must be known. However, the correspondences are not given for the linear methods, since - as mentioned above - it is the actual task of the nonlinear registration method to estimate the dense point correspondences. Theoretically, landmark-based methods can estimate the linear component correctly, given dense landmarks. Establishing such landmarks however, is hardly feasible in practice, and would render the nonlinear registration step superfluous. A very similar discussion is presented in Yezzi and Soatto [12].

Statistical Shape Models. Statistical Shape Models (SSM) are an emerging technique in the field of medical image processing and analysis. SSMs are supposed to capture the information about shape variations of a certain population. SSMs are not only a valuable tool for studying shape variations of organs and pathologies, but also provide a mean to capture prior knowledge and thus aid the process of nonlinear registration in the demanding inter-patient scenario, in which standard methods may easily fail. For creation of SSMs, two different approaches are used: landmark- and deformation-based. For the landmark-based approach, corresponding landmarks have to be computed for all images of the population, compare e.g. Cootes *et al.* [1]. This can be a very challenging and time-consuming task. Thus, the idea was developed to create models of shape using deformations which result from nonlinear registration, compare e.g. Grenander and Miller [5], Gee and Bajcsy [3], Joshi [7], Rueckert *et al.* [10], or Cootes *et al.* [2].[2]

With respect to important properties of deformation

[2] In the context of computing shape models from deformations, the term *Statistical Deformation Models (SDM)* is sometimes also used [10]. For generality, we employ the term *Statistical Shape Models (SSM)* throughout this paper.

fields for construction of SSMs, the literature agrees on the fact that the fields should be free of any similarity transformation, that is, they should not contain any amount of scaling, rotation and translation, compare for example [10, Sec II.A, p. 1016] or [2, Sec. 4, p. 452]. This is a very intuitive goal, since these parameters do not describe a variation in shape between single subjects of a population, but a variation in pose and size. Usually, it is supposed that the global preregistration accounts for this transformation. However, due to the reasons stated above, in general, linear registration methods cannot fully recover linear transformations in deformable scenarios, which leads to deformation fields which contain a linear transformation component. Thus, using these deformation fields presents a potential source of error for construction of SSMs. The goal of the presented work is to eliminate this source of error by computing deformation fields containing a minimal amount of similarity transformation.

Computation of Minimal Deformations. The proposed method decomposes a given deformation into a similarity transformation and a minimal nonlinear deformation part. The minimality of the nonlinear part is computed with respect to the mean squared Euclidean norm of the displacement field representing the deformation. The actual computation is performed by using a closed-form solution. The minimization problem is modeled in such way that the computed minimal deformation is expressed in the reference frame of the target image. Since the method operates on point correspondences, it can be applied to dense deformation fields in the complete image domain as well as only to a region of interest. The method is comparably fast and since it presents a post-processing step for any given point correspondences, it can be easily integrated into any existing framework for construction of shape models based on deformations.

Contribution We consider the following points to be the major contribution of the presented work.

1. We study the performance of linear registration methods in presence of nonlinear deformations, and show that for real data, after the global preregistration, there is a significant linear component of the transformation which is not retrieved.
2. We propose a method to a posteriori extract the similarity transformation component from any given deformation field.
3. We theoretically show that using non-minimal deformations for constructing shape models can be expected to result in models which do not describe shape variations appropriately.
4. We empirically confirm the results of the theoretical analysis, and show that the removal of similarity components from deformation fields prior to model construction leads to improved shape models in terms of interpretability and compactness.

2. Methods

2.1. Notation and Basic Definitions

In the context of registration, the transformation which aligns the target and source images I_T and I_S is a function $T : \Omega \rightarrow \Omega$ where $\Omega \subset \mathbb{R}^d$ is the image domain of dimension $d = 2, 3$. In most of the current methods for deformable scenarios, the transformation T is composed of a global, linear transformation T_{global} and the nonlinear local part T_{local}, resulting in

$$T = T_{global} \circ T_{local} \quad , \tag{1}$$

where $\circ$ denotes composition. In general, the global part is computed prior to the local component and no joint computation of the two terms is employed.

We model nonlinear transformations as a sum of the identity function Id and a displacement field U, as $T = \mathrm{Id} + U$. For the local nonlinear transformation we also apply the notation $T_{local}(X) = Y$.

2.2. Computation of Minimal Deformations

Our goal is to extract the remaining linear transformation component from a given deformation. To this end, we model the deformation as a composition of a linear and a nonlinear part

$$T_{local} = T_{lin} \circ T_{nl} \quad . \tag{2}$$

The task now is to estimate T_{lin} and T_{nl}, such that T_{nl} becomes minimal in some meaningful sense.

The model from Eq. (2) can be reformulated, such that it allows us to minimize the norm of the displacement field U_{nl} of the nonlinear component T_{nl} with respect to the linear transformation T_{lin}.

$$\begin{aligned} T_{local}(X) &= (T_{lin} \circ T_{nl})(X) && (3)\\ Y &= T_{lin}(X + U_{nl}(X)) && (4)\\ T_{lin}^{-1}(Y) - X &= U_{nl}(X) \quad . && (5) \end{aligned}$$

Here, in (4) we use $T_{local}(X) = Y$, and express the deformation T_{nl} by the displacement U_{nl}.

Thus, we can define a cost function, the optimization of which results in a linear transformation (described by parameters p), such that the norm of the vectors of the displacement field becomes minimal with respect to the mean squared norm. The cost function E for the displacement fields discretized by n points is given by

$$E(X, Y, p) = \frac{1}{n} \sum_i \left\| X_i - T_{lin}^{-1}(Y_i; p) \right\|^2 \quad , \tag{6}$$

and the respective minimization is

$$p = \arg\min_{p'} E(X, Y, p') \ . \tag{7}$$

In other words, the extraction of any other linear transformation would result in larger displacements (given the reference frame and the type of linear transformation).

We chose to use the squared norm for several reasons. First, this model has an exact and fast closed-form solution. Second, for the application to SSMs, the squared norm is the most common choice in literature, and it is consistent with the PCA-based shape model.

Once the linear transformation T_{lin}^{-1} is computed, the corresponding displacement field is resulting from Eq. (5) as $U_{nl}(X) = T_{lin}^{-1}(Y; p) - X$, and the nonlinear remaining part can be constructed by $T_{nl} = \mathrm{Id} + U_{nl}$.

Please note that the minimal deformation T_{nl} is expressed in the reference frame of the target image. This is an important property for the application of the method to SSMs, compare [2]. Please note also that our method does not change the results of the complete registration procedure, but rather computes a different decomposition, that is $T = T_{global} \circ T_{local} = T_{global} \circ T_{lin} \circ T_{nl}$.

Up to this point, the discussion is valid for any invertible linear transformation T_{lin}. In the following we constrain the discussion to a similarity transformation.

2.2.1 Least-Squares Optimization

For the computation of the similarity transformation which minimizes the mean squared norm of the displacement field, we employ the closed-form solution of Umeyama [11], which is shown to give the exact result.

It solves the so called *Absolute Orientation Problem*, which consists of finding the similarity transformation which minimizes the mean squared distance between two point sets A and B of arbitrary dimension d, that is

$$e^2(R, t, c) = \frac{1}{n} \sum_{i=1}^{n} \|B_i - (cRA_i + t)\|^2 \quad , \tag{8}$$

where c is the scaling factor, t is the translation vector, and R is the rotation matrix, and n is the number of points. For space reasons, the details of the implementation are given in Sec. A of the supplementary material, available at `http://campar.in.tum.de/personal/zikic/cvpr2008/`.

We can apply this method to our problem directly by identifying B with points X of the image domain, which are the origins of the vectors of the displacement field, and identifying A with $Y = X + U(X)$, that is, the destination points of the displacement vectors. The computed entities R, t, c are used to parametrize the similarity transformation T_{lin}^{-1}.

Since the minimization operates on a set of two corresponding point sets, the method is not restricted to dense deformations, but can be applied to arbitrary point sets. For SSMs, a meaningful choice is to restrict the computation to regions of interest of the deformation fields [10].

The computation is comparably fast with the complexity of $\mathcal{O}(dn + d^3)$ and has a memory consumption of $\mathcal{O}(dn + d^2)$. The complexities are linear in n and as we have $d << n$, this makes the method very attractive for our application. In practice, the runtimes of our MATLAB implementation are for example 0.13s for a 2D example with $n = 100^2$ points and 6.27s for $n = 100^3$ in 3D.

2.3. Statistical Shape Models

Principal component analysis (PCA) is the preferred method for statistical shape models [10, 2]. The attractive properties of the PCA for shape modeling include optimal linear reconstruction of the data set variance, the estimated modes of variation are orthogonal and uncorrelated, and a closed form solution exists for calculating the principal components at a relatively low computational cost.

The shape model is built from m given displacement fields $U = \{U_i\}$ representing the deformations. The d-dimensional deformation fields with n displacement vectors are linearized as column vectors $u_i \in \mathbb{R}^{dn}$.

From u_i, a linear shape model, which approximates a given field u is given by $\bar{u}$ and Φ as

$$u = \bar{u} + \Phi b \ . \tag{9}$$

Here $\bar{u}$ is the mean of all m displacement fields, that is $\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_i$. The matrix Φ is constructed from the k first eigenvectors Φ_i of the covariance matrix C, given by $C = \frac{1}{m-1}\sum_{i=1}^{m}(u_i - \bar{u})(u_i - \bar{u})^\top$. The eigenvalues corresponding to Φ_i are denoted by λ_i. The vectors Φ_i are also referred to as *modes*. Finally, $b \in \mathbb{R}^k$ is the parameter vector, describing the contribution of the principal modes contained in Φ in order to approximate u by the employed linear model. By assuming a Gaussian distribution on the single displacements entries, the variance of the parameter b_i can be given by λ_i [10].

An important measure for evaluating the constructed model is the so called *reconstruction error* e_{rec}, given by

$$e_{rec}(\bar{u}, \Phi, u, b) = \|u - (\bar{u} + \Phi b)\|^2 \ . \tag{10}$$

The reconstruction error measures the error between a given vectorized displacement field u, and the reconstruction of u by using the parameters b corresponding to u, and the model given by $\bar{u}$ and Φ. The parameter vector b is computed by a projection of u onto the model, that is by $b = \Phi^\top(u - \mu_u)$, where μ_u denotes the mean of u. The variance explained by a single mode Φ_i corresponds to the variance of b_i, which is $\mathrm{Var}\left[\Phi_i^\top(u - \mu_u)\right]$.

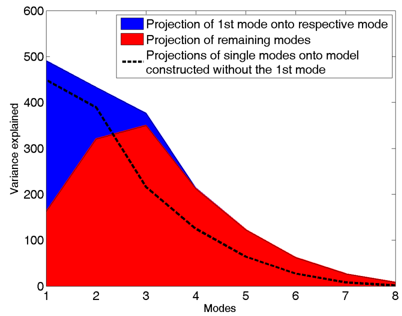


Figure 2: Example demonstrating how the largest mode used for generation of data (blue area) is mixed with the remaining modes (red). We measure this by the amount of the explained variance. This behavior is due to the finite number of samples used, in which case the PCA does not reconstruct the actual modes which generate the data, but rather their linear combination. The black curve shows the true variance of the remaining modes (2nd to last).

With respect to the proposed method, the only modification of the standard model construction process is that instead of the original displacement fields U, we use minimal deformation fields U_{nl}, from which the maximum amount of similarity transformation is extracted by our method as described in Sec. 2.2.

2.3.1 Influence of Similarity Transformations in Deformation Fields on SSMs

In this Section, we argue that if similarity transformation components are not removed from the deformation fields, this will in general lead to shape models in which the first modes do not necessarily describe the largest variations in shape. For a more detailed derivation of this argument, please refer to Sec. B of the supplementary material.

It is a general property of the PCA that - when operating on a finite number of samples - it does not compute the actual modes which generate the data, but rather a linear combination of these. This behavior is illustrated in Fig. 2 for a general example. Here, a model is constructed from $n = 122$ orthogonal samples. It can be seen how the variance which is actually generated by the first mode during construction of the samples is explained by the first four reconstructed modes.

In particular, this general behavior also occurs for data, which can be seen as generated as a combination of nonlinear deformation and similarity transformations. This is the case for non-minimal deformations. This means that the similarity transformations will not be represented by single modes, but their contribution is distributed over several modes describing nonlinear deformation. Since similarity transformations are global, the corresponding modes have a large variance, such that mostly the first modes of the model will be influenced by the effect described above. As a consequence, for non-minimal deformations, in general we can not expect that the first modes of the model describe the largest variations in shape. This underlines the importance of using minimal deformation fields. Our experimental tests on real data (Fig. 4 and Fig 5) closely resemble the behavior predicted in Fig. 2.

Figure 3: Three example instances of the Deformed Spoon.

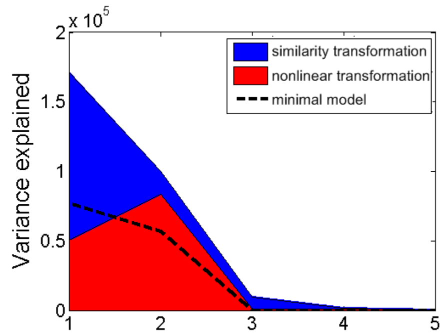


Figure 4: Analysis of the single modes of the original shape model on the synthetic spoon data set (deformations by NRM 1). The variance explained by single modes of the respective model is given. The contribution for the original model is decomposed into the variance explaining the similarity transformation component and the nonlinear component.

3. Results and Evaluation

In the following, we discuss and evaluate the results of the application of the proposed method to the construction of shape models.

3.1. Test Settings

In this section we briefly present the synthetic and the corpus callosum setting, which are used for evaluation.

3.1.1 Synthetic Example

To demonstrate some of the propositions of the presented work on a simple example, a synthetic data set with ground truth deformations is constructed. The set consists of 100 deformed versions of an image of a spoon - a selection is illustrated in Fig. 3. The spoons are deformed randomly according to two deformation modes, one altering the cup-size and the other controlling the grip width. The outline of the spoons in the target image is equipped with a dense set of 50 equidistant landmarks. The transformations used for warping the images are also applied to the landmarks. The deformed spoons are registered by a similarity transformation which minimizes the squared norm of the errors on the landmarks.

3.1.2 Test on Real Corpus Callosum Data

We test the proposed method on real data which is part of the LADIS (Leukoaraiosis And DISability) study [9], a pan-European study involving 12 hospitals and more than 600 patients. The data in question consists of 62 2D MR images of the midsagittal cross-section of the corpus callosum brain structure. The data set is equipped with a set of 72 corresponding landmarks in each image chosen by physicians. Attention is payed in order to achieve a possibly accurate and well-distributed set of landmarks. For an example image of the data set, please refer to Fig. 1a.

For the creation of SSMs on the corpus callosum data, nonlinear registration is performed by two different methods after a linear preregistration. While the first method (NRM 1) is dedicated and tested for the creation of shape models, the second (NRM 2) is an alternative method used for comparison in Sec. 3.2.

Linear Preregistration. The linear preregistration step is performed by a similarity transformation based on the landmarks in the images, minimizing the squared norm.

Nonlinear Registration. The nonlinear registration method (NRM 1) primarily used for computation of the deformation fields is dedicated to the construction of shape models and is shown to generate accurate results for the corpus callosum data in question [6].

The method computes the deformations $T_{local\,i}$, based on registering the images I_i of the data set to the reference image I_R, by solving the following minimization problem iteratively

$$T_{local} = \min_{T'_{local}} \sum_i \mathcal{D}(I_R, I_i \circ T'_{local\,i}) + \mathcal{S}(T'_{local\,i}) \ , \quad (11)$$

where $\mathcal{D}$ is a similarity measure between images, and $\mathcal{S}$ is a regularization term on the transformation. The process is iterative in the sense that starting from an initial estimate of the reference image I_R, which is simply an average of all the images, a new, improved estimate is computed in every iteration until convergence. It can be shown that under an assumption of Gaussian distribution of the noise, this choice of reference image is optimal with respect to achieving an unbiased coordinate frame for the shape model [8]. For the corpus callosum data, $\mathcal{D}$ is chosen as sum of squared distances and $\mathcal{S}$ as the L2 norm on the parameter space. The parameterization of the deformation model is performed by free-form deformation (FFD), based on sine-kernels.

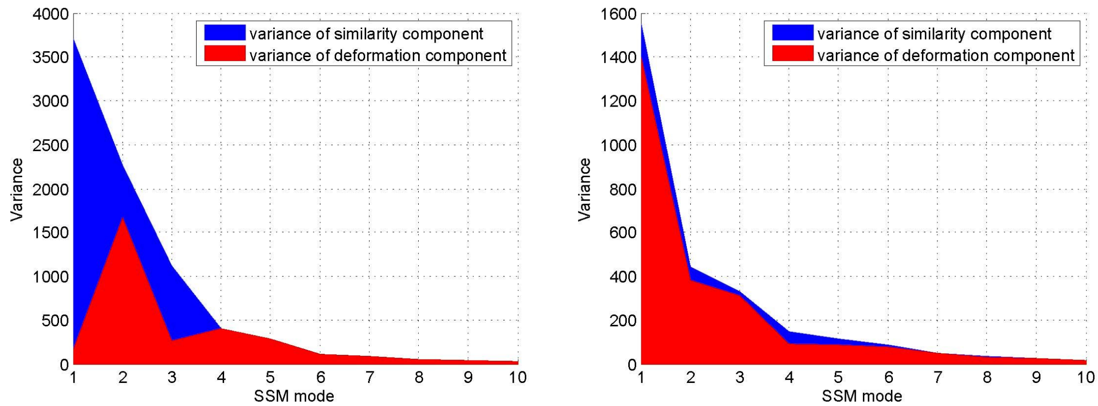
Nonlinear Registration II. This alternative method (NRM 2) is used in Sec. 3.2, in order to demonstrate that the occurrence of similarity components in deformation fields is independent of the chosen nonlinear method. The method is based on B-spline FFDs and uses discrete optimization using Markov random fields. It differs from NRM 1 in choice of parameters, grid resolution, and the optimization method. For details, please refer to [4].

Method	Param	μ_a	m_a	σ_a	$\max_a$	μ	σ
NRM 1 (dense)	α	0.816	0.609	0.773	5.224	0.053	1.123
	Δc	0.028	0.022	0.025	0.165	-0.001	0.037
	t_x	2.08	1.51	2.03	12.46	-1.08	2.70
	t_y	2.23	1.58	2.20	12.93	-1.07	2.95
NRM 1 (region)	α	1.361	0.972	1.357	10.112	0.090	1.920
	Δc	0.028	0.016	0.034	0.191	-0.002	0.044
	t_x	2.61	1.83	2.74	18.54	-1.07	3.63
	t_y	2.60	1.66	2.86	19.47	-1.02	3.73
NRM 1 (boundary)	α	1.293	0.967	1.272	9.312	0.080	1.813
	Δc	0.028	0.015	0.034	0.187	-0.001	0.044
	t_x	2.60	1.84	2.69	18.21	-1.07	3.59
	t_y	2.55	1.64	2.83	19.37	-0.99	3.69
NRM 2 (dense)	α	0.698	0.588	0.556	3.955	0.294	0.843
	Δc	0.010	0.008	0.008	0.056	0.001	0.013
	t_x	1.28	1.15	0.91	6.02	1.03	1.19
	t_y	1.39	1.23	0.95	5.24	-1.10	1.28
NRM 2 (region)	α	1.458	1.158	1.365	9.034	0.321	1.972
	Δc	0.019	0.011	0.022	0.157	-0.003	0.028
	t_x	2.10	1.46	2.04	12.80	0.84	2.81
	t_y	2.07	1.58	2.06	13.36	-1.05	2.72
NRM 2 (boundary)	α	1.408	1.159	1.241	8.226	0.254	1.860
	Δc	0.018	0.011	0.021	0.150	-0.002	0.027
	t_x	2.07	1.53	1.92	11.76	0.81	2.70
	t_y	2.01	1.54	1.94	12.25	-0.96	2.62

Table 1: Quantification of the similarity transformations extracted for the corpus callosum data set. The *amount* of the transformation is described by the mean (μ_a), median (m_a), standard deviation (σ_a) and maximum ($\max_a$) of the norm of the parameters. The *variation* is given by the mean (μ) and standard deviation (σ) of the actual parameters. The scaling is expressed as deviation from 1, that is $\Delta c = c - 1$. The rotation is given in degrees, translation in millimeters, and scaling is a unit-less factor.

3.2. Quantification of Extracted Similarity Transformation Components

In this section, we quantify the amount of similarity transformations which were extracted from deformations computed for the corpus callosum example. We describe



(a) Analysis of standard SSM

(b) SSM built on minimal displacements

Figure 5: Analysis of single modes of SSMs built on displacements from the corpus callosum data set (results based on NRM 2 (region)). We evaluate the contributions of the similarity transformation and the actual deformation to the variance of each mode, by decomposing the modes into respective components (areas represent component contributions). **(a)** illustrates an SSM constructed from the original deformations, for which several modes capture more information about similarity transformation than shape. In **(b)**, we observe a significant improvement achieved by building the SSM on minimal displacements. The remaining similarity is due to the approximate orthogonality of similarity and minimal displacement components.

the *amount* of the extracted similarity transformations by computing the mean, median, standard deviation, and maximum of the norm of the computed parameters. The *variation* of the parameters is described by the mean and standard deviation of the actual parameters (not their norm).

In order to show that the existence of the effect is not dependent on the nonlinear registration method, we computed the results by (NRM 1) as well as by an alternative method (NRM 2) described in 3.1.2.

Furthermore, the extraction is performed by considering three different regions of the image domain: 1) the complete *dense* field on the whole image domain, 2) only the segmented *region* of the corpus callosum on the reference image, and 3) only the *boundary* of the corpus callosum on the reference image. As discussed in [10], the construction of the model based on the region of interest can be a meaningful choice for a specific part of anatomy. For these cases (*region*, *boundary*), the extracted similarity components are larger than for the complete dense field, compare Table 1.

The results in Table 1 demonstrate that there is a large and highly varying amount of similarity transformation contained in the computed deformation fields. For a visualization of the extracted similarity transformations (based on the deformation in the corpus callosum region), please refer to the supplementary material.

3.3. Effects of Proposed Method on SSMs

In this section we discuss the impact of using minimal deformations for construction of shape models and compare the resulting models with original SSMs. We demonstrate that the modes of the original SSM contain a substantial amount of similarity transformation. Creation of SSM from minimal deformations largely reduces this negative effect. We also performed an analysis of the reconstruction ability of the SSMs constructed by the proposed method. The results show a slightly improved relative reconstruction error for the SSMs created on minimal deformations. However, since the reconstruction ability is not the focus of this work, the results are presented in Sec. C of the supplementary material.

To gain further insight into the reconstruction ability of the models, the single modes of the model are examined. For this, the modes of the original model are divided into a similarity and a nonlinear component, as described in Sec. 2.2.1. Then, the variance is assigned to the single components relative to their squared norms, based on the assumption that the components are nearly orthogonal. In Figs. 4 and 5 we visualize and analyze the variance explained by the single components of the original model, and compare this with the modified model. The results correspond to the theoretical prediction given in Sec. 2.3.1 and Fig. 2.

The most important observation is that the first modes of the original model contain a large similarity transformation component. Actually, in all our experiments, the first mode contains mainly a similarity transformation component. This means that if non-minimal deformations are used to construct the model, the first mode does not describe the strongest variation in shape. The proposed method removes a large amount of similarity transformation from the resulting SSM, which leads to a more compact model, in which every mode mainly describes shape, compare Fig. 5.

The remaining amount of similarity in the SSM is due to the fact that the displacement fields representing the nonlinear and linear components computed as described in Sec. 2.2 are not orthogonal. Since the similarity transformations form a manifold and not a linear subspace in the space of all deformations, a completely orthogonal decomposition is not possible. Our experiments show that the proposed method can be seen as an approximately orthogonal decomposition which leads to only a small amount of similarity transformations in modes of the resulting SSM.

4. Discussion and Conclusion

In this work, we show that similarity transformations are inherently a part of the deformation fields obtained by standard nonlinear registration schemes. This is in contrast to the assumptions usually made. We propose a method which computes minimal nonlinear deformations by extracting the similarity transformation components.

We show that using minimal deformations is crucial for the construction of shape models, since otherwise the models are seen to describe other effects than change in shape. Particularly, the proposed method eliminates the negative effect that the first modes of a shape model do not necessarily describe the largest amount of shape, which can occur for SSMs based on non-minimal deformations.

Registering landmark sets to a joint reference frame prior to model construction is common practice for shape models based on landmarks, and our approach can be seen as transferring this concept to the case of deformation-based SSMs.

Existing SSM frameworks based on deformations can be extended in a straight forward way to make use of the proposed method and thus benefit from using minimal deformations.

5. Acknowledgements

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