Complex Systems- Exercises 9

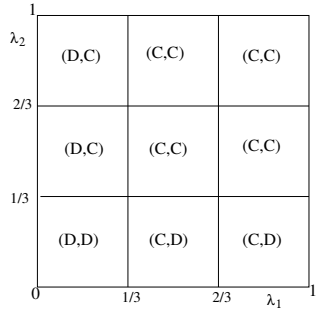
1. Check that the statements in the lecture notes on the Nash equilibria and the Pareto optimal strategies for the games of Prisoner's Dilemma, Battle of Sexes, Stag Hunt, Chicken, and Matching Pennies as stated in the lecture notes are correct.

Solution. Routine.

2. Consider the Prisoner's Dilemma with the payoff matrix.

$(1_{\downarrow}, 2 \rightarrow)$	C	D
C	(3,3)	(0,4)
D	(4,0)	(1,1)

Given the associated social/moral game with $m = 2$, find the Nash equilibria for all the other social/moral coefficients in the unit square $[0, 1] \times [0, 1]$ for the double game of Prisoner's Dilemma as in the notes. You need to find the Nash equilibria at the boundaries of two or more regions as well.



Solution. In the main 9 regions the Nash equilibria are as in the above diagram. At every boundary point of these regions, the set of Nash equilibria is the union of all Nash equilibria in the main neighbouring regions. For example, for $\lambda_1 = 1/3$ and $0 \leq \lambda_2 < 1/3$, the set is $\{(D, D), (C, D)\}$, whereas for $\lambda_1 = 1/3$ and $\lambda_2 = 1/3$, the set is $\{(D, D), (C, D), (D, C), (C, C)\}$ (every strategy profile is a Nash equilibrium), and for $\lambda_1 = 2/3$ and $\lambda_2 = 1/3$, the set is $\{(C, D), (C, C)\}$.