233 Computational Techniques

Tutorial 1

Problem 1. Countable sets. Check the following results.

(i) The set of all positive integers is countable.

(ii) The set of all integers is countable.

(iii) We can show by induction on n that the set of ordered lists of natural numbers that have length n is countable.

(iv) We can then use (iii) to show that the set of all finite ordered lists of natural numbers is countable.

(v) Any non-finite subset of a countable set is countable.

(vi) If S is countable then S^n, i.e., the collection of all n-tuples of elements of S, is countable.

(vii) From (vi), we can deduce that the set of integer polynomials (i.e., polynomials with integer co-efficients) is countable.

(viii) From (vii) it follows that the set of roots of integer polynomials, the so-called algebraic numbers, is also countable.

Problem 2.
Accuracy of floating point operations. Assuming rounded binary arithmetic, determine *(a)* the supremum of values $\delta > 0$ such that $fl(1+\delta) = 1$, and *(b)* the number of significant decimal digits, for both single and double precision.

Problem 3.
Error propagation in the arithmetic operations. Analyze the propagation of the relative error in each of the four arithmetic operations by comparing the relative error of the result with the sum of the relative errors of the operands, assuming that the operations themselves do not introduce additional loss of accuracy. (Treat multiplication first, it's the easiest!)